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New model for the neutrino mass matrix

L. Lavoura

Universidade Técnica de Lisboa

Centro de Física das Interacções Fundamentais

Instituto Superior Técnico, 1049-001 Lisboa, Portugal

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Abstract

I suggest a model based on a softly broken symmetry $L_e - L_\mu - L_\tau$ and on Babu's mechanism for two-loops radiative generation of the neutrino masses. The model predicts that one of the physical neutrinos (ν_3) is massless and that its component along the ν_e direction (U_{e3}) is zero. Moreover, if the soft-breaking term is assumed to be very small, then the vacuum oscillations of ν_e have almost maximal amplitude and solve the solar-neutrino problem. New scalars are predicted in the 10 TeV energy range, and a breakdown of e–μ–τ universality should not be far from existing experimental bounds.

In a model without right-handed (singlet) neutrinos, the three weak-interaction-eigenstate neutrinos ν_e, ν_μ, and ν_τ may acquire $|\Delta I| = 1$ Majorana masses given by the following term in the Lagrangian:

$$\mathcal{L}^{(\nu)}_{\rm mass} = \frac{1}{2} \left(\begin{array}{ccc} \nu_e^T & \nu_\mu^T & \nu_\tau^T \end{array} \right) C^{-1} \mathcal{M} \left(\begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \right) - \frac{1}{2} \left(\begin{array}{ccc} \overline{\nu_e} & \overline{\nu_\mu} & \overline{\nu_\tau} \end{array} \right) C \mathcal{M}^* \left(\begin{array}{c} \overline{\nu_e}^T \\ \overline{\nu_\mu}^T \\ \overline{\nu_\tau}^T \end{array} \right). \tag{1}$$

Here, C is the Dirac–Pauli charge-conjugation matrix and $\mathcal{M}$ is a 3×3 symmetric mass matrix. One may diagonalize $\mathcal{M}$ with help of a unitary matrix U in the following way:

$$U^T \mathcal{M} U = \mathrm{diag}\,(m_1, m_2, m_3)\,, \tag{2}$$

where m_1, m_2, and m_3 are real and non-negative. The physical neutrinos ν_1, ν_2, and ν_3 are given by

$$\left(\begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \right) = U \left(\begin{array}{c} \nu_1 \\ \nu_2 \\ \nu_3 \end{array} \right). \tag{3}$$

Then,

$$\mathcal{L}^{(\nu)}_{\rm mass} = \frac{1}{2} \sum_{i=1}^{3} m_i \left(\nu_i^T C^{-1} \nu_i - \overline{\nu_i} C \overline{\nu_i}^T \right). \tag{4}$$

Experiment indicates that two linearly independent squared-mass differences among the three physical neutrinos differ by a few orders of magnitude. Indeed, $\Delta m^2_{\rm atm}$ is of order $10^{-3}\,\mathrm{eV}^2$, while $\Delta m^2_\odot$ may be either of order $10^{-5}\,\mathrm{eV}^2$, in the case of the MSW solution for the solar-neutrino puzzle, or of order $10^{-10}\,\mathrm{eV}^2$, in the case of the vacuum-oscillations ("just so") solution. It is customary to identify ν_3 as the neutrino which has a mass much different from the masses of the other two, *viz.*,

$$\left| m_2^2 - m_1^2 \right| = \Delta m^2_\odot \ll \left| m_3^2 - m_1^2 \right| \approx \left| m_3^2 - m_2^2 \right| \approx \Delta m^2_{\rm atm}. \tag{5}$$

Then, the negative result of CHOOZ's search for ν_e oscillations [1] is interpreted as $|U_{e3}| \leq 0.217$, which is valid for $\Delta m^2_{\rm atm} \geq 2 \times 10^{-3}\,\mathrm{eV}^2$.

It has been pointed out [2] that the assumption of an approximate lepton-number symmetry $\bar{L} \equiv L_e - L_\mu - L_\tau$ (where L_e is the electron number, L_μ is the muon number, and L_τ is the tau number) may constitute a good starting point for a model of the neutrino mass matrix. Indeed, if there are no $\left|\Delta \bar{L}\right| = 2$ mass terms then

$$\mathcal{M} = \left(\begin{array}{ccc} 0 & rb & b \\ rb & 0 & 0 \\ b & 0 & 0 \end{array} \right), \tag{6}$$

where b and r may, without loss of generality, be taken to be real and positive. The mass matrix in Eq. (6) yields $m_3 = 0$, $m_1 = m_2 = b\sqrt{1+r^2}$, and

$$U = \left(\begin{array}{ccc} \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \\ \frac{r}{\sqrt{2\left(1+r^2\right)}} & \frac{ir}{\sqrt{2\left(1+r^2\right)}} & \frac{1}{\sqrt{1+r^2}} \\ \frac{1}{\sqrt{2\left(1+r^2\right)}} & \frac{i}{\sqrt{2\left(1+r^2\right)}} & -\frac{r}{\sqrt{1+r^2}} \end{array} \right). \tag{7}$$

This is good for the following reasons:

1. The negative result of CHOOZ's search for ν_e oscillations gets explained through $U_{e3} = 0$.

2. Since $4\left|U_{e1}U_{e2}\right|^2 = 1$, vacuum oscillations of ν_e with maximal amplitude would occur were $m_1 \neq m_2$, opening way for the "just so" solution of the solar-neutrino problem to apply.

3. It is intuitive to expect r to be close to 1. Now, if $r = 1$ then ν_μ–ν_τ mixing is maximal, and this explains the atmospheric-neutrino anomaly.

On the other hand, $\bar{L}$ must be broken, because $m_1 = m_2$ does not allow for oscillations between ν_1 and ν_2 and a solution of the solar-neutrino puzzle. A good choice, in order to avoid unpleasant majorons, would be to have $\bar{L}$ to be softly broken; this would moreover permit a natural explanation for $\Delta m^2_\odot \ll \Delta m^2_{\mathrm{atm}}$. This option has been suggested by Joshipura and Rindani [3]; however, in those authors' models there is no predictive power for the form of the mixing matrix U, a fact which impairs the immediate interest and experimental testability of those models.

In this paper I put forward a simple model with softly broken $\bar{L}$ which maintains some predictive power. The model is based on Babu's mechanism for two-loops radiative generation of the neutrino masses [4]. I remind that, in general, Babu's mechanism leads to one neutrino remaining massless; however, whereas that general mechanism cannot predict the ν_e, ν_μ, and ν_τ components of the massless neutrino, the specific model that I shall put forward retains the exact-$\bar{L}$ prediction $U_{e3} = 0$. Moreover, in my model there is a rationale for the ν_e oscillations of maximal amplitude, and for the tiny mass difference $\Delta m^2_\odot$, which allow a "just so" explanation of the solar-neutrino deficit; that rationale is provided by the naturalness of the assumption that the term which breaks $\bar{L}$ softly is very small.

In my model I just introduce in the scalar sector, above and beyond the usual standard-model doublet $\phi = \left(\begin{array}{cc} \varphi^+ & \varphi^0 \end{array} \right)^T$, one singly-charged singlet f^+ with $\bar{L} = 0$, together with two doubly-charged singlets g^{2+} and h^{2+}, and their Hermitian conjugates. The difference between g^{2+} and h^{2+} lies in that the former field has $\bar{L} = 0$ whereas h^{2+} has $\bar{L} = -2$. The Yukawa couplings of the leptons are $\bar{L}$-invariant and are given by

$$\begin{aligned} \mathcal{L}_{\mathrm{Y}}^{(1)} &= -\frac{m_e}{v} \left(\begin{array}{cc} \overline{\nu_{eL}} & \overline{e_L} \end{array} \right) \left(\begin{array}{c} \varphi^+ \\ \varphi^0 \end{array} \right) e_R - \frac{m_\mu}{v} \left(\begin{array}{cc} \overline{\nu_{\mu L}} & \overline{\mu_L} \end{array} \right) \left(\begin{array}{c} \varphi^+ \\ \varphi^0 \end{array} \right) \mu_R - \frac{m_\tau}{v} \left(\begin{array}{cc} \overline{\nu_{\tau L}} & \overline{\tau_L} \end{array} \right) \left(\begin{array}{c} \varphi^+ \\ \varphi^0 \end{array} \right) \tau_R \\ &\quad + f^+ \left[f_\mu \left(\nu_{eL}^T C^{-1} \mu_L - e_L^T C^{-1} \nu_{\mu L} \right) + f_\tau \left(\nu_{eL}^T C^{-1} \tau_L - e_L^T C^{-1} \nu_{\tau L} \right) \right] \\ &\quad + e_R^T C^{-1} \left[g^{2+} \left(g_\mu \mu_R + g_\tau \tau_R \right) + h^{2+} h_e e_R \right] + \mathrm{H.c.}, \end{aligned} \tag{8}$$

where f_μ, f_τ, g_μ, g_τ, and h_e are complex coupling constants. Notice that, in the first line of Eq. (8), I have already taken, without loss of generality, the Yukawa couplings of ϕ to be flavor-diagonal; v denotes the vacuum expectation value of φ^0.

The scalar potential V has a trivial part, V_{trivial}, which is a quadratic polynomial in $\phi^\dagger\phi$, f^-f^+, $g^{2-}g^{2+}$, and $h^{2-}h^{2+}$. Besides, V includes two other terms, with complex coefficients λ and ϵ:

$$V = V_{\text{trivial}} + \left(\lambda f^- f^- g^{2+} + \epsilon g^{2-} h^{2+} + \text{H.c.}\right). \tag{9}$$

The term with coefficient ϵ breaks $\bar{L}$ softly. I make the following assumptions: this is the only $\bar{L}$-breaking term in the theory, and ϵ is small. These assumptions are technically natural in the sense of 't Hooft [5].[1]

From now on I shall assume, without loss of generality, f_μ, f_τ, g_τ, h_e, λ, and ϵ to be real and positive. Only g_μ remains, in general, complex.

The neutrino mass term $\mathcal{M}_{e\mu}$ does not break $\bar{L}$ and is generated at two-loops level by the Feynman diagram in Figure 1. A similar diagram generates $\mathcal{M}_{e\tau}$. In both cases, there is in the diagram an inner charged lepton which may be either μ or τ. It is clear that the mass terms thus generated obey the relation

$$r \equiv \frac{\mathcal{M}_{e\mu}}{\mathcal{M}_{e\tau}} = \frac{f_\mu}{f_\tau}. \tag{10}$$

Contrary to what happens in Zee's model [6], this ratio of mass terms is not proportional to a ratio of squared charged-lepton masses [7]. As seen before, in order to obtain maximal ν_μ–ν_τ mixing one would like to have $r \approx 1$. In the present model, this means that the coupling constants f_μ and f_τ should be approximately equal. In Zee's model, on the other hand, one winds up with the rather unrealistic constraint $f_\mu/f_\tau \approx (m_\tau/m_\mu)^2$.

Let us check whether the diagram in Figure 1 is able to yield neutrino masses of the right order of magnitude. As we shall see later, we would like to obtain $|\mathcal{M}_{e\mu}| \approx |\mathcal{M}_{e\tau}| \approx \sqrt{\Delta m^2_{\text{atm}}} \sim 10^{-2}$–$10^{-1}\,\text{eV}$. Now, from the diagram in Figure 1 with an inner τ one obtains

$$\mathcal{M}_{e\mu} = -2\lambda f_\mu f_\tau g_\tau m_e m_\tau \frac{I}{(16\pi^2)^2}, \tag{11}$$

where

$$\begin{aligned} I &= \frac{1}{\pi^4}\int d^4k \, \frac{1}{k^2 - m_f^2}\,\frac{1}{k^2 - m_e^2}\int d^4q \,\frac{1}{q^2 - m_f^2}\,\frac{1}{q^2 - m_\tau^2}\,\frac{1}{(k-q)^2 - m_g^2} &(12)\\ &= \frac{1}{2\left(m_f^2 - m_\tau^2\right)}\int_0^\infty \frac{dy}{(y+1)(y+x_e)}\left[p\ln\frac{y+x_g+1+p}{y+x_g+1-p} - p'\ln\frac{y+x_g+x_\tau+p'}{y+x_g+x_\tau-p'}\right.\\ &\quad \left. + (1-x_\tau)\ln x_g + (x_\tau - x_g - y)\ln x_\tau\right]. &(13)\end{aligned}$$

Here, $x_e = m_e^2/m_f^2$, $x_\tau = m_\tau^2/m_f^2$, $x_g = m_g^2/m_f^2$, and

$$\begin{aligned} p &= \sqrt{(y+x_g-1)^2 + 4y}, &(14)\\ p' &= \sqrt{(y+x_g-x_\tau)^2 + 4yx_\tau}. &(15)\end{aligned}$$

The integral in Eq. (13) is convergent and may be computed numerically.[2] For m_e, $m_\tau \ll m_f$ and $m_g \approx m_f$, one finds I to be of order m_f^{-2}.

In my estimate of $\mathcal{M}_{e\mu}$ I shall therefore set $I \approx m_f^{-2}$. The bounds from e–μ–τ universality in μ decay and in τ decay are $f_\mu/m_f \lesssim 10^{-4}\,\text{GeV}^{-1}$ and $f_\tau/m_f \lesssim 10^{-4}\,\text{GeV}^{-1}$ [4]; if one allows $f_\mu f_\tau/m_f^2$ to be as high as $10^{-8}\,\text{GeV}^{-2}$, then one obtains

$$|\mathcal{M}_{e\mu}| \approx 10^{-15}\lambda g_\tau. \tag{16}$$

It is reasonable to assume that the Yukawa coupling g_τ is of the same order of magnitude as the Yukawa couplings f_μ and f_τ, and that the dimensionful scalar-potential coupling constant λ is of the same order

[1] Notice that the possible $\bar{L}$-breaking term $f^-f^-h^{2+}$ has dimension higher than the one of $g^{2-}h^{2+}$, and therefore the assumption of its absence is natural.

[2] It is not possible to use the approximations $m_e = m_\tau = 0$ because they lead to infrared divergences. This is not a problem, since those divergences are logarithmic and $\mathcal{M}_{e\mu}$ in Eq. (11) also includes a factor $m_e m_\tau$.

of magnitude as both m_f and m_g. This leads to $g_\tau/\lambda \sim f_\mu/m_f \sim 10^{-4}\,\mathrm{GeV}^{-1}$. Fortunately the product λg_τ stays free. In order to obtain $|\mathcal{M}_{e\mu}| \sim 10^{-2}\,\mathrm{eV}$ it is then sufficient to assume

$$\lambda \approx m_g \approx m_f \quad \sim \quad 10^4\,\mathrm{GeV}, \tag{17}$$

$$f_\mu \approx f_\tau \approx g_\tau \quad \sim \quad 1. \tag{18}$$

Extra factors of order 1 may easily enhance $|\mathcal{M}_{e\mu}|$ and bring it up to the desired value 0.06 eV.

The assumption, made in Eq. (18), that the Yukawa couplings are of order 1, may seem unrealistic.[3] However, there are no experimental indications against this possibility when the masses of f^+ and of g^{2+} are assumed to be as high as 10 TeV.[4] For instance, g^{2+} mediates the unobserved decay $\tau^- \to \mu^- e^+ e^-$; however, by comparing that decay with the standard $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$, one easily reaches the conclusion that $\mathrm{BR}\left(\tau^- \to \mu^- e^+ e^-\right)$ should be at least one order of magnitude below the present experimental bound, when $m_g \approx 10\,\mathrm{TeV}$ and $|g_\mu g_\tau| \approx 1$. A more complicated process is $e^+e^- \to \tau^+\tau^-$, which is mediated by g^{2+} exchange in the t channel. The amplitude A for this process is

$$\begin{aligned} A \;=\; & \frac{ie^2}{s}\left[\bar{v}(e)\gamma^\mu u(e)\right]\left[\bar{u}(\tau)\gamma_\mu v(\tau)\right] + \frac{ie^2}{3\left(s-m_z^2\right)}\left[\bar{v}(e)\gamma^\mu\gamma_5 u(e)\right]\left[\bar{u}(\tau)\gamma_\mu\gamma_5 v(\tau)\right] \\ & - \frac{ig_\tau^2}{8\left(t-m_g^2\right)}\left[\bar{v}(e)\gamma^\mu\left(1+\gamma_5\right)u(e)\right]\left[\bar{u}(\tau)\gamma_\mu\left(1+\gamma_5\right)v(\tau)\right]. \end{aligned} \tag{19}$$

I have used the convenient approximations $m_e = m_\tau = 0$ and $\sin^2\theta_w = 1/4$ in writing down the standard-model amplitude, and a Fierz transformation in the non-standard contribution. If one defines $j = 2m_g^2/s$, $z = g_\tau^2/\left(2e^2\right)$, and $l = 3\left(s-m_z^2\right)/s$, then one finds

$$\frac{d\sigma}{d\cos\theta} \propto \frac{l^2+1}{l^2}\left(1+\cos^2\theta\right) + \frac{4}{l}\,\cos\theta + z\frac{l+1}{l}\frac{\left(1+\cos\theta\right)^2}{1+j+\cos\theta} + z^2\frac{\left(1+\cos\theta\right)^2}{\left(1+j+\cos\theta\right)^2}, \tag{20}$$

where θ is the angle between the momenta of e^- and of τ^- in the center-of-momentum frame. From the differential cross section in Eq. (20) one easily checks that the deviations of both the total cross section and the forward–backward asymmetry from their standard-model predictions are completely negligible when $m_g \sim 10\,\mathrm{TeV}$, even if g_τ is as large as 1.

Except for $\mathcal{M}_{e\mu}$ and $\mathcal{M}_{e\tau}$, all other matrix elements of $\mathcal{M}$ break $\bar{L}$ and, therefore, they will all be proportional to the $\bar{L}$-breaking parameter ϵ, which is assumed to be small. The matrix elements $\mathcal{M}_{\mu\mu}$, $\mathcal{M}_{\mu\tau}$, and $\mathcal{M}_{\tau\tau}$ arise at two loops from the diagram in Figure 2. In order to obtain a non-zero $\mathcal{M}_{ee}$ one must go to three loops and use for instance the diagram in Figure 3. In that diagram there are two inner charged leptons which may be either μ or τ; therefore, there is a contribution to $\mathcal{M}_{ee}$ proportional to m_τ^2, and that matrix element should not be neglected in spite of it only arising at three-loops level.

The diagram in Figure 2 clearly leeds to the following relation:

$$\mathcal{M}_{\mu\mu} : \mathcal{M}_{\mu\tau} : \mathcal{M}_{\tau\tau} = f_\mu^2 : \left(f_\mu f_\tau\right) : f_\tau^2 = r^2 : r : 1. \tag{21}$$

One thus obtains that in the present model

$$\mathcal{M} = \left(\begin{array}{ccc} a & rb & b \\ rb & r^2c & rc \\ b & rc & c \end{array}\right), \tag{22}$$

where a, b, and c are complex numbers with mass dimension, while $r = f_\mu/f_\tau$ is a real dimensionsless number which should in principle be of order 1. The masses a and c are suppressed relative to b by the soft-breaking parameter ϵ.

The mass matrix in Eq. (22) immediately leads to two predictions of this model: there is one massless neutrino (ν_3) and its component along the ν_e direction, *i.e.*, U_{e3}, vanishes. Indeed, the diagonalizing

[3]Notice however that, in the standard model, the top-quark Yukawa coupling is also very close to 1.

[4]Concerns about the breakdown of perturbativity are only justified for Yukawa couplings $\gtrsim 4\pi$, *i.e.*, of order 10 or more.

matrix U reads

$$U = \begin{pmatrix} \cos\psi & -i\sin\psi & 0 \\ e^{i\alpha}\dfrac{r\sin\psi}{\sqrt{1+r^2}} & e^{i\alpha}\dfrac{ir\cos\psi}{\sqrt{1+r^2}} & \dfrac{1}{\sqrt{1+r^2}} \\ e^{i\alpha}\dfrac{\sin\psi}{\sqrt{1+r^2}} & e^{i\alpha}\dfrac{i\cos\psi}{\sqrt{1+r^2}} & -\dfrac{r}{\sqrt{1+r^2}} \end{pmatrix} . \operatorname{diag}\left(e^{i\theta_1}, e^{i\theta_2}, 1\right), \tag{23}$$

cf. Eq. (7). In the matrix of Eq. (23) $\alpha \equiv \arg\left[ab^* + bc^*\left(1+r^2\right)\right]$ is a physically meaningless phase. The Majorana phases θ_1 and θ_2 are necessary in order to obtain real and positive m_1 and m_2. The sole physically observable phase is $2\left(\theta_1 - \theta_2\right)$ [8]. The mixing angle ψ is given by

$$\tan\psi = \sqrt{1+\varepsilon^2} + \varepsilon, \tag{24}$$

where

$$\varepsilon = \frac{|c|^2\left(1+r^2\right) - |a|^2}{2\sqrt{1+r^2}\,|ab^* + bc^*\left(1+r^2\right)|} \tag{25}$$

is a parameter of order ϵ, just as a/b and c/b, and may therefore be assumed to be very small. Thus, ψ is close to $45°$. The amplitude of the vacuum oscillations of ν_e relevant for the solution of the solar-neutrino problem is $4\left|U_{e1}U_{e2}\right|^2 = \left(1+\varepsilon^2\right)^{-1}$, *i.e.*, almost maximal. Thus, the present model favors a "just so" solution of the solar-neutrino puzzle.

The soft-breaking parameter ϵ should be tiny. Indeed, one finds

$$\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} \approx 2\frac{\left|ab^* + bc^*\left(1+r^2\right)\right|}{|b|^2\sqrt{1+r^2}} \sim \epsilon; \tag{26}$$

as we want the "just so" solution for the solar-neutrino puzzle to apply, we must accept ϵ to be of order 10^{-7}. Such a tiny soft breaking of $\bar{L}$ may eventually be explained by some new physics at a very high energy scale.

From the non-observation of neutrinoless double beta decay one derives the bound $|\mathcal{M}_{ee}| \leq 0.2\,\mathrm{eV}$ [9]. This is not a problem to the present model. Indeed, as m_3 is predicted to vanish, m_1 and m_2 should both be very close to $\sqrt{\Delta m^2_{\rm atm}} \approx 0.06\,\mathrm{eV}$. Thus, in the approximation $\cos^2\psi = \sin^2\psi = 1/2$, one has

$$|\mathcal{M}_{ee}| \approx (0.03\,\mathrm{eV})\left|e^{2i(\theta_1-\theta_2)} - 1\right| < 0.2\,\mathrm{eV}. \tag{27}$$

Moreover, the phase $2\left(\theta_1 - \theta_2\right)$ is very close to zero—indeed, it vanishes in the limit of $\bar{L}$ conservation.

In conclusion, the model that I have presented in this paper makes the exact predictions $m_3 = 0$ and $U_{e3} = 0$, while it naturally accomodates maximal amplitude ν_e oscillations and a tiny $\Delta m^2_\odot$. Maximal ν_μ–ν_τ mixing follows from the reasonable assumption that two Yukawa couplings are almost equal. Neutrino masses are small because they are radiatively generated at two-loops level. Indeed, the fact that two neutrino masses are as *large* as $0.06\,\mathrm{eV}$ practically forces the new mass scale, at which the extra scalars lie, to be in the $10\,\mathrm{TeV}$ range; while deviations from e–μ–τ universality in μ decay and in τ decay should be close at hand. The model requires some physical mechanism for generating a tiny soft breaking of $\bar{L}$.

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Figure captions

Figure 1: Two-loops Feynman diagram which generates $\mathcal{M}_{e\mu}$.

Figure 2: Two-loops Feynman diagram which generates $\mathcal{M}_{\mu\tau}$.

Figure 3: One of the three-loops Feynman diagrams which generate $\mathcal{M}_{ee}$.

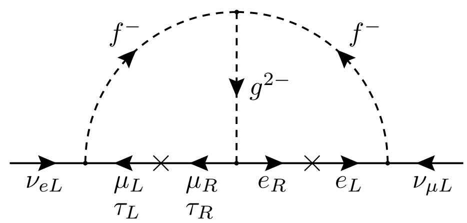

Figure 1

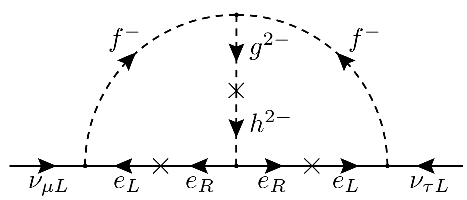

Figure 2

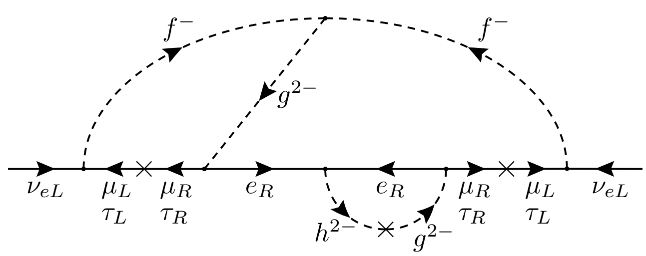

Figure 3